Emergence of communities in social networks

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Emergence of communities in social networks?

- Model of large social networks with focus on how communities emerge
- Model should reproduce characteristic properties AND communities
- Start from large-scale empirical social network

J.-P. Onnela, J. Saramäki, J. Hyvönen, G. Szabó, D. Lazer, K. Kaski, J. Kertész, and A.-L. Barabási, PNAS 104, 7332 (2007).

J. M. Kumpula, J.-P. Onnela, J. Saramäki, K. Kaski, and J. Kertész, Phys. Rev. Lett. 99, 228701 (2007).

Overview

1. Social networks
2. Empirical social network
3. Modelling social networks
4. Conclusion

Social networks

- Social network paradigm in the social sciences: Social life consists of the **flow** and exchange of norms, values, ideas, and other social and cultural resources **channelled through the social network**

- Perspective:
  - Focus on very large networks
  - Focus on statistical properties
  - Complex networks & statistical mechanics

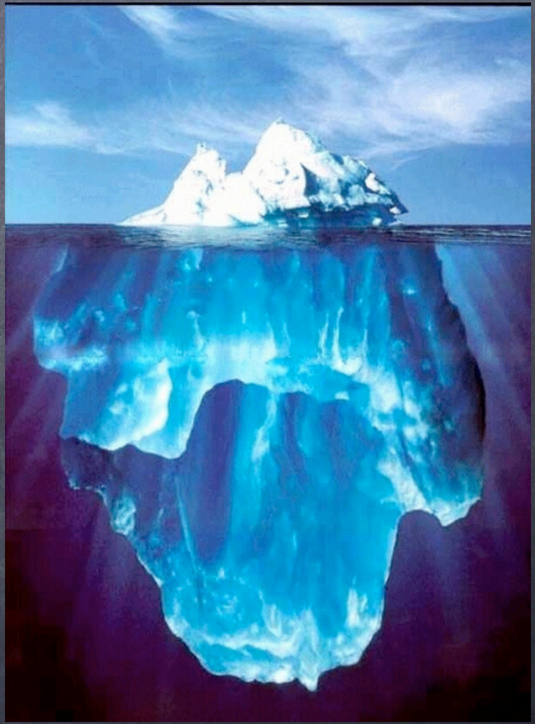

Photo from http://defiant.corban.edu/gtipton/net-fun/iceberg.html

Social networks

- Traditional approach:
  - Data from questionnaires; $N \approx 10^2$
  - Scope of social interactions wide
  - Strength based on recollection

- New approach:
  - Electronic records of interactions; $N \approx 10^6$
  - Scope of social interactions narrower
  - Strength based on measurement

COMPLEMENTARY APPROACHES

- Constructed network is a proxy for the underlying social network

1. Social networks

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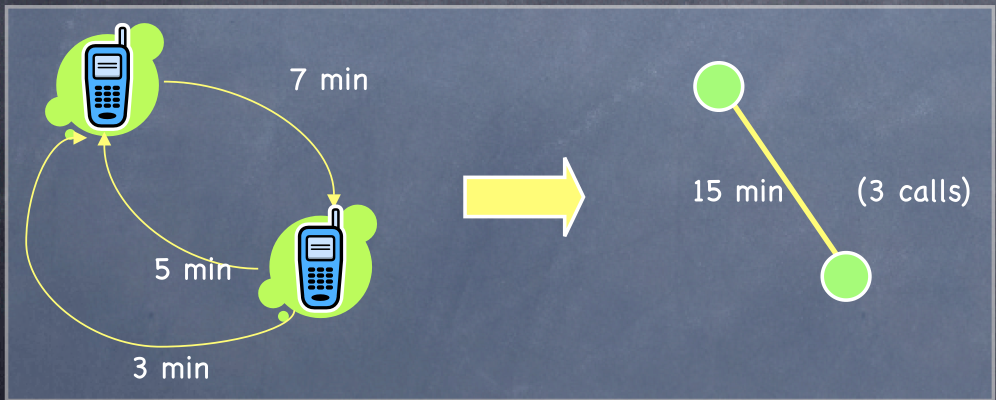
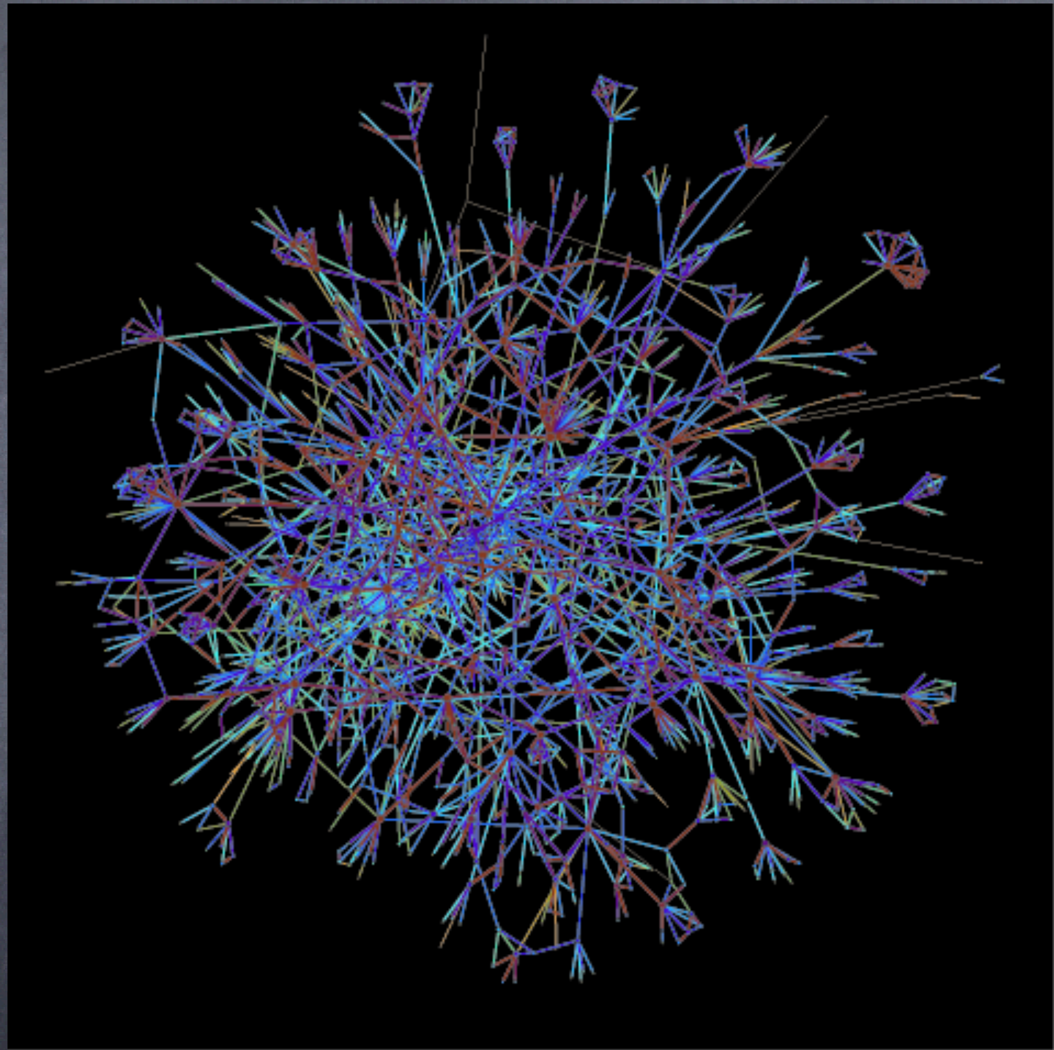
Constructing empirical network

- Data
  - One operator in a European country, 20% coverage
  - Aggregated from a period of 18 weeks
  - Over 7 million private mobile phone subscriptions
  - Voice calls within the operator
- Require reciprocity of calls for a link
- Quantify tie strength (link weight)

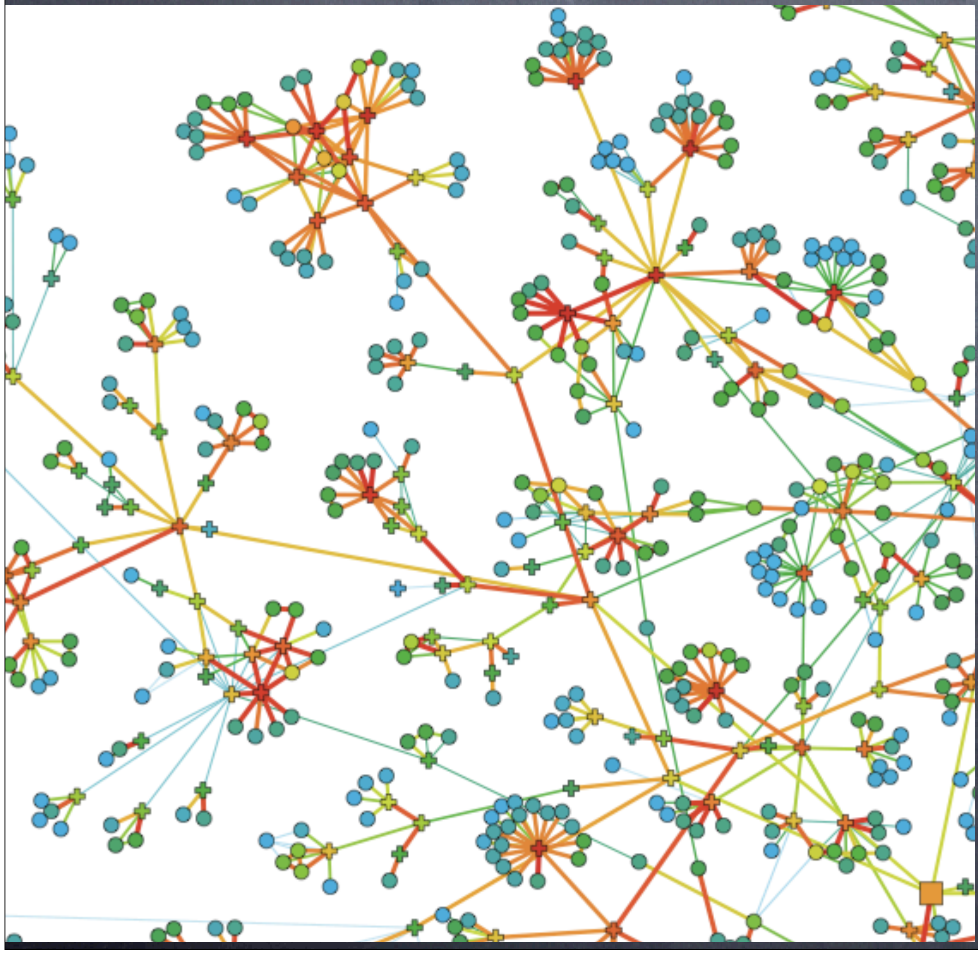


Aggregate call duration w_{ij}^{D}

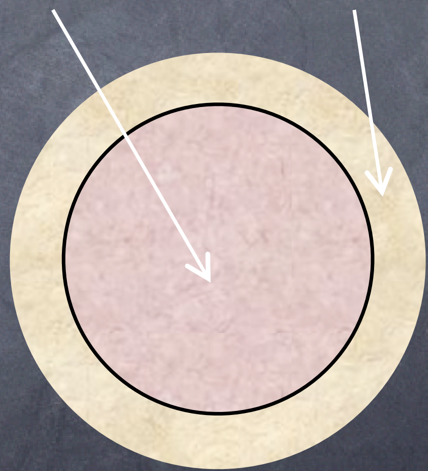
Total number of calls w_{ij}^{N}



About (social) network visualisation

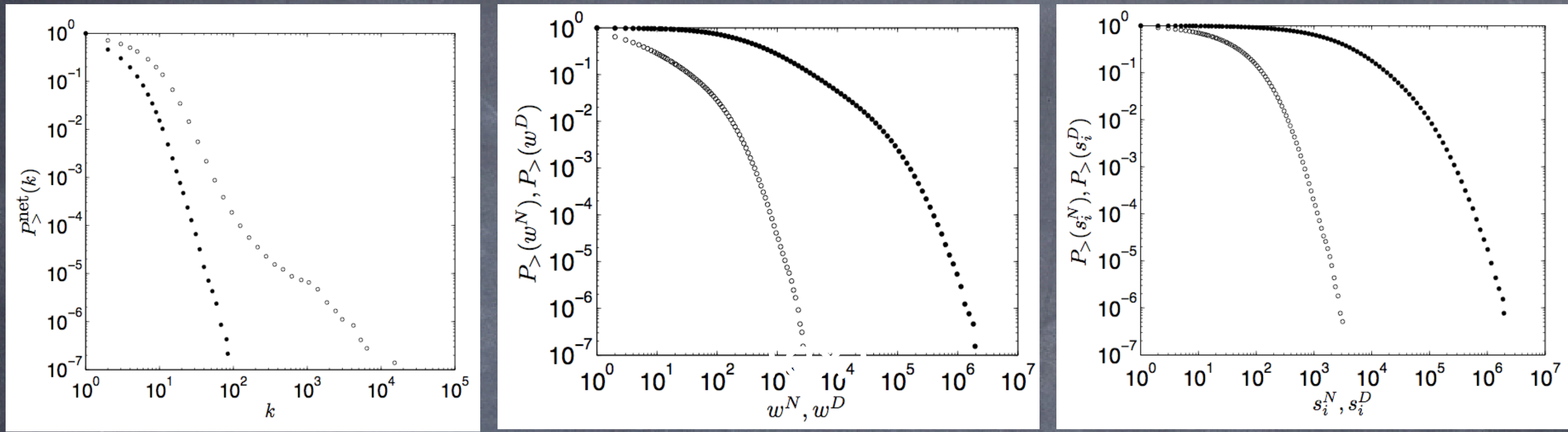


- Snowball sampling (distance!)
- Bulk nodes & surface nodes



- Majority are surface nodes
- Neighbour visibility

Network statistics

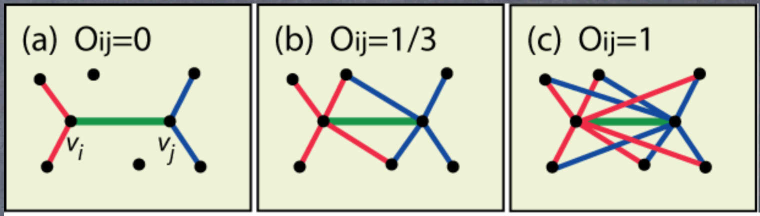


	mean	std	max
degree k	3.3	2.5	144
weight w^N	15.4	37.3	3,610
weight w^D	41 min	206 min	663 h
strength s^N	51	75	3,644
strength s^D	135 min	386 min	690 h

degree = # of links

Local structure

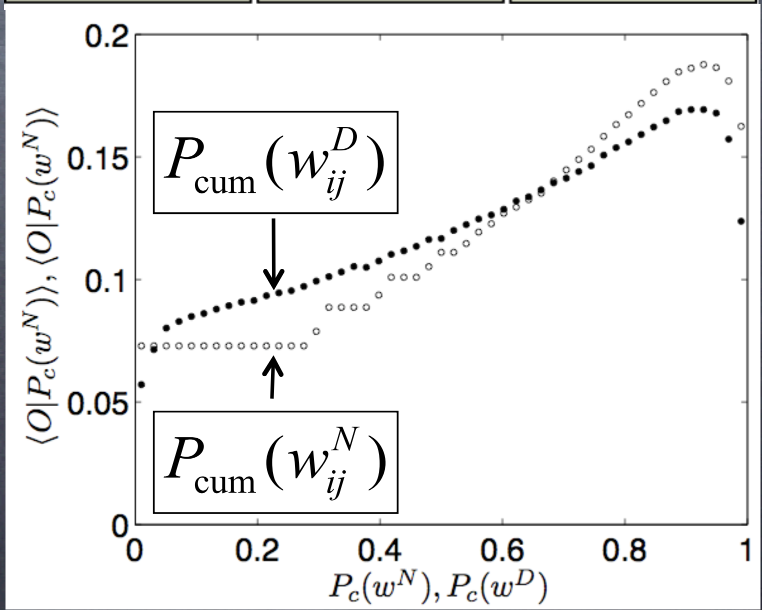
$$O_{ij} = \frac{n_{ij}}{(k_i - 1) + (k_j - 1) - n_{ij}}$$



- Weak ties hypothesis*: Relative overlap of two individual's friendship networks varies with the strength of their tie to one another

- Define **overlap** O_{ij} of edge (i,j) as the fraction of common neighbours

- Average overlap increases as a function of (cumulative) link weights



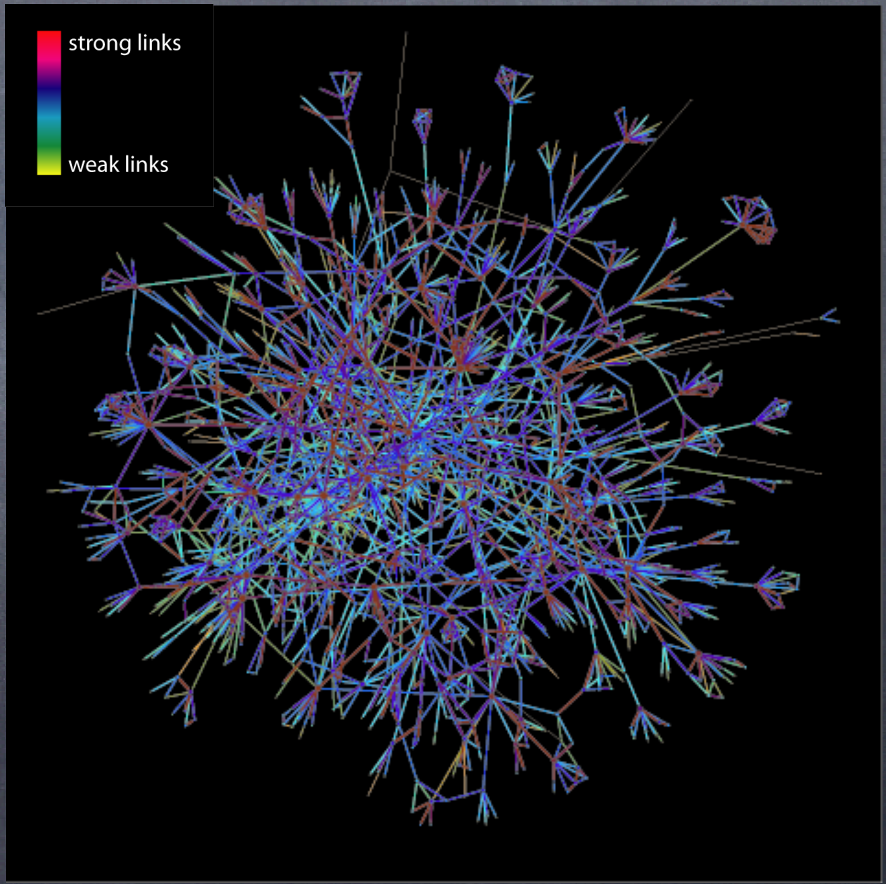
* M. Granovetter, The strength of weak ties, AJS **78**, 1360 (1973)

Global structure

- Probe the global role of links of different weight and local topology
- Approach of physicists (and children): Break to learn!
- Thresholding (percolation): Remove links based on their weight
- Control parameter f is the fraction of removed links
    - Initial network (f=0); isolated nodes (f=1)

Global structure

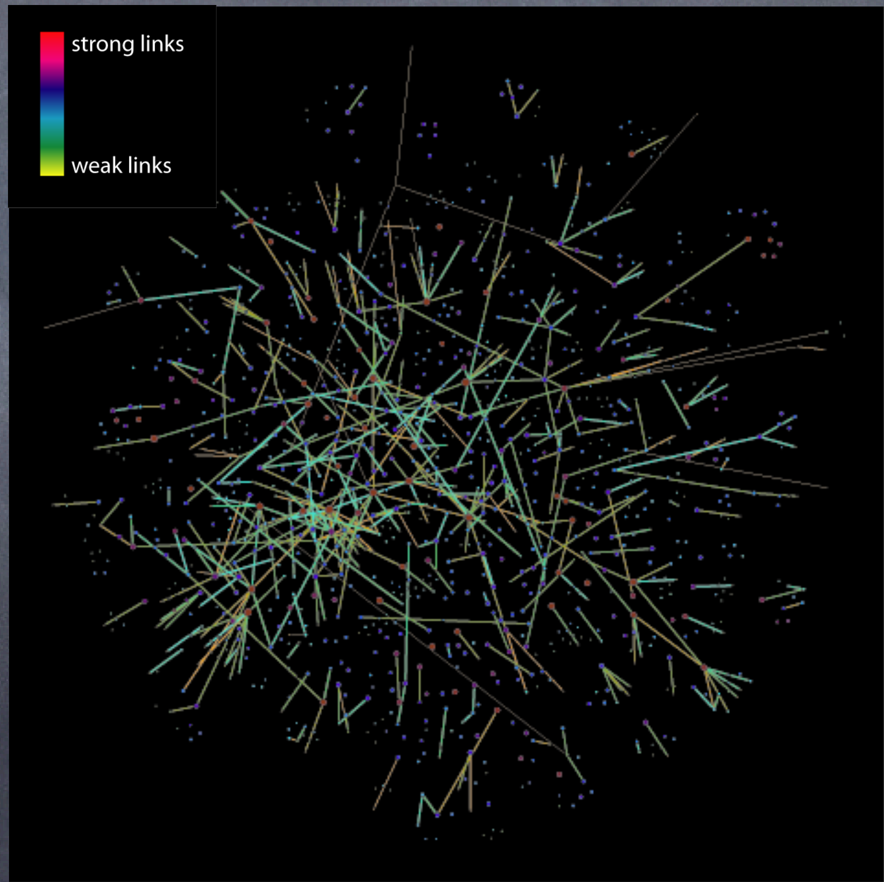
Initial connected network (f=0), small sample
⇒ All links are intact, i.e. the network is in its initial stage



Global structure

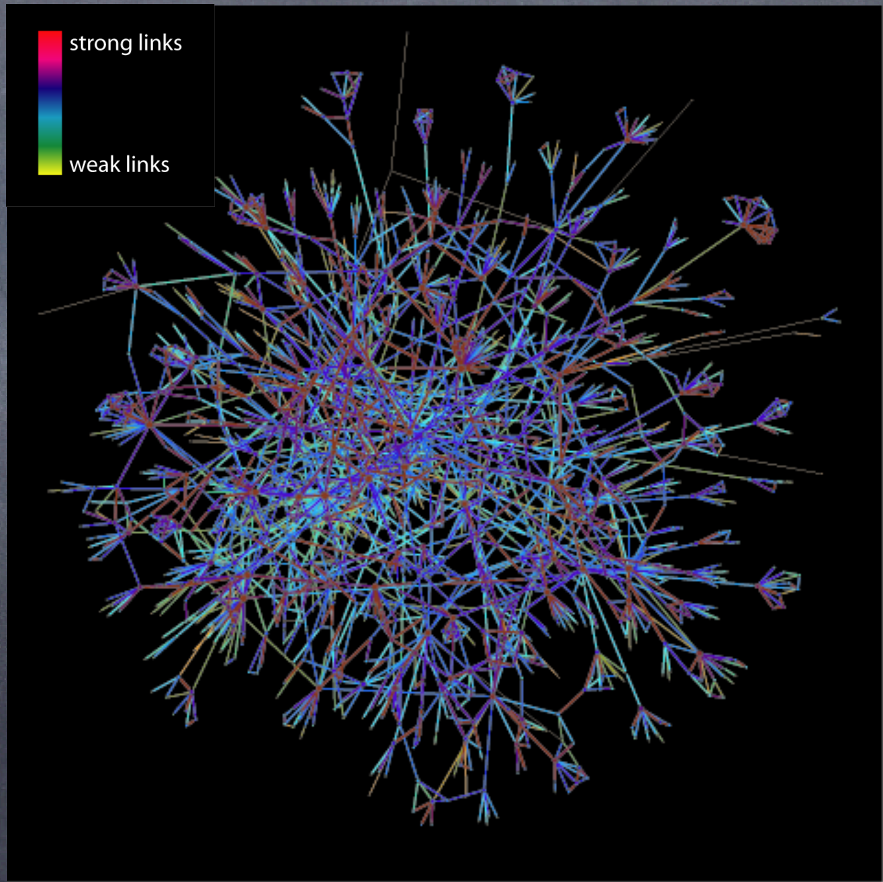
Decreasing weight thresholded network (f=0.8)

⇒ 80% of the strongest links removed, weakest 20% remain



Global structure

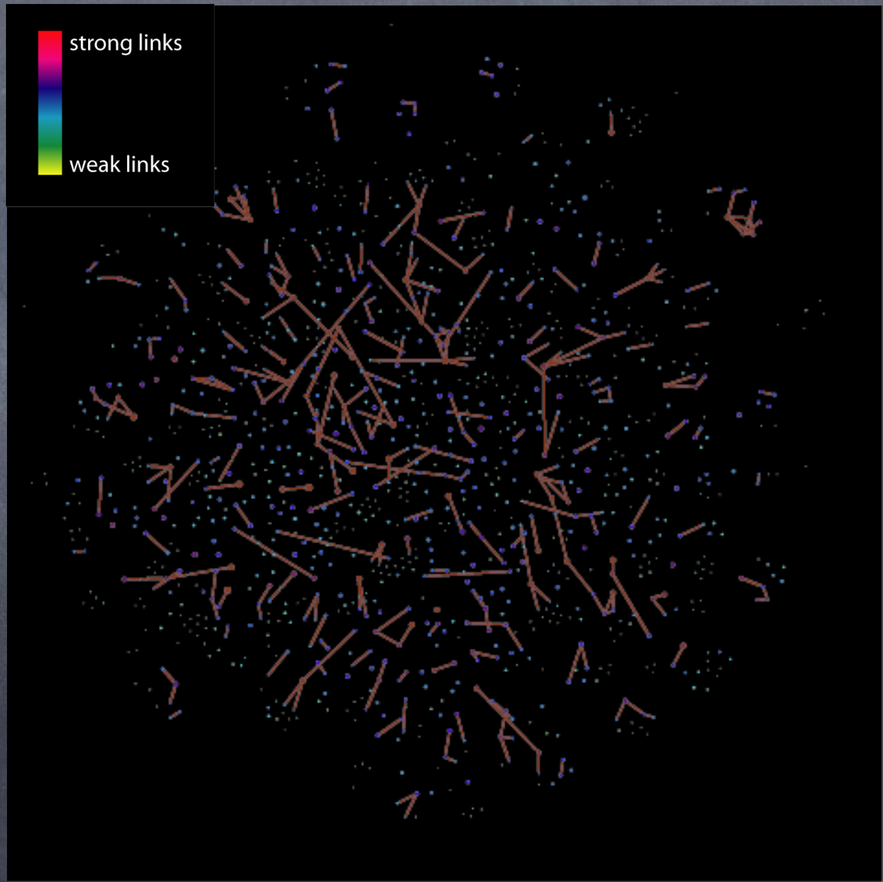
Initial connected network (f=0), small sample
⇒ All links are intact, i.e. the network is in its initial stage



Global structure

Increasing weight thresholded network (f=0.8)

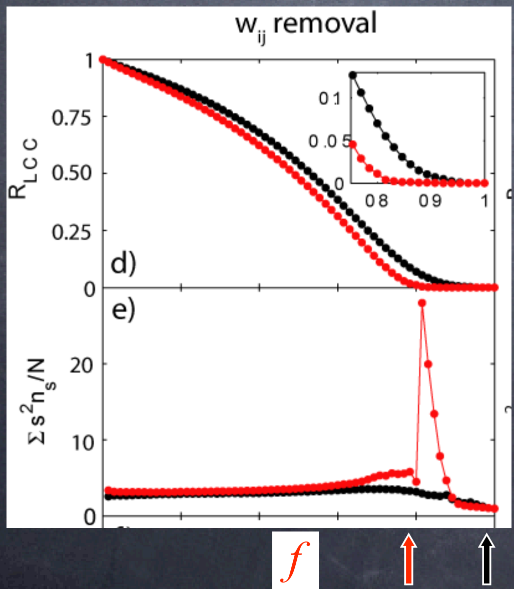
⇒ 80% of the weakest links removed, strongest 20% remain



Global structure

- **Qualitative** difference in the **global** role of weak and strong links
  - Phase transition when weak ties are removed first $f_c(\infty) \neq 1$
  - No phase transition when strong ties are removed first $f_c(\infty) = 1$
- Suggests a point of division between weak and strong links (f_c)

$$w_c = P^{-1}_{\text{cum}}(0.80) \approx 27\,\text{min}$$



$f_c^w(\infty) = 0.80 \pm 0.04$

"globally connected" phase
"disconnected islands" phase

Order parameter R_{LCC}
- Def: fraction of nodes in LCC

Susceptibility S
- Def: average cluster size (excl. LCC)

$$\left[S = \sum_{s<s_{\max}} n_s s^2 / \sum_{s<s_{\max}} n_s s;\ \tilde{S} = \sum_{s<s_{\max}} n_s s^2 / N;\ C_i = t_i / 2k_i(k_i - 1) \right]$$

Summary of empirical study

- Communities have mostly strong ties within (WTH)
- Communities are interconnected mostly with weak ties (percolation)

1. Social networks

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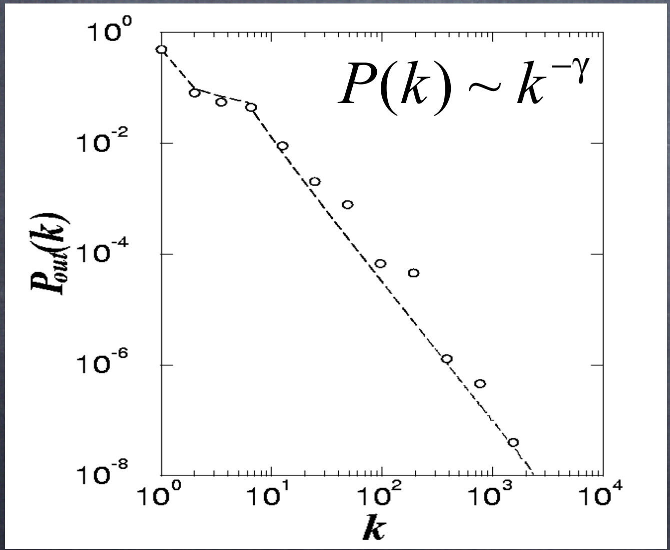
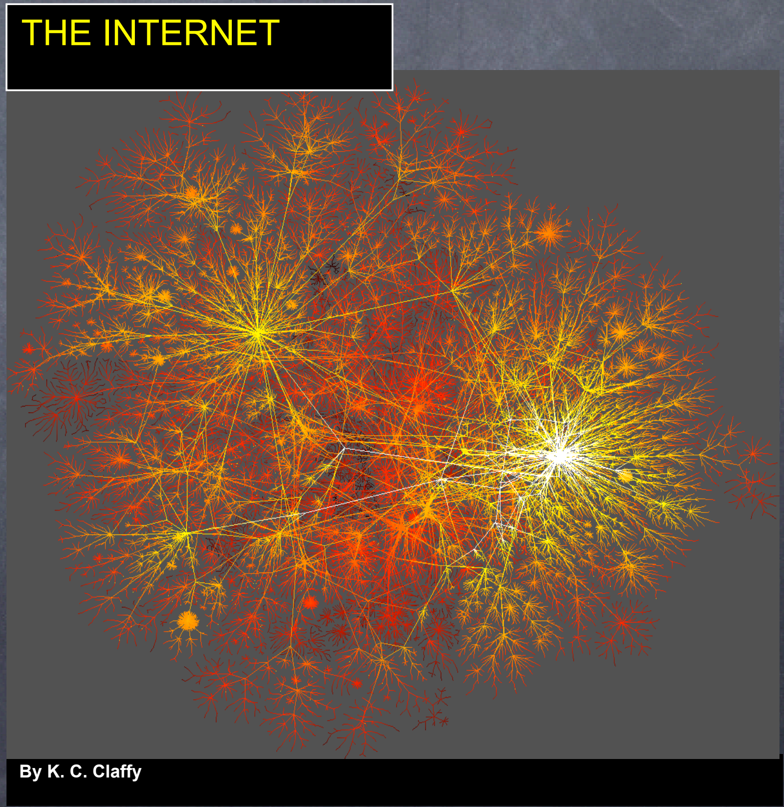
Intro to modelling

- Social networks appear to have some “universal features”
- Can these features be reproduced with a simple microscopic model?
- Network sociology: How individual microscopic interactions translate into macroscopic social systems
- Statistical mechanics: How individual microscopic interactions translate into macroscopic (physical) systems

Intro to modelling

- Internet & web => Simple rules work





Intro to modelling

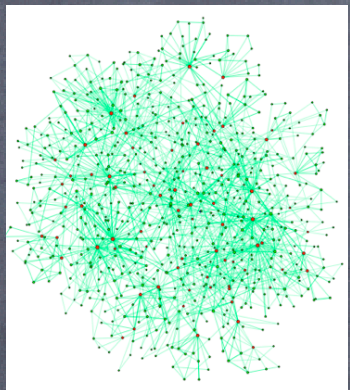
- A weighted model of social networks with focus on emergence of communities (mesoscopic structures) from microscopic rules
- Fixed number of nodes N
- Aim to reproduce characteristics features, no fitting to data
- Regression models in sociology
- No claim for a grand unified theory of social networks

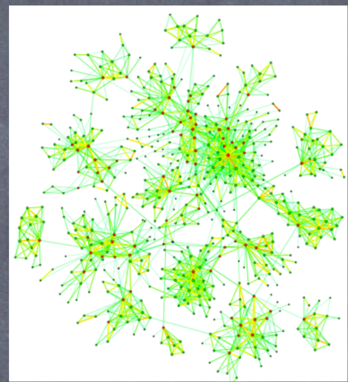
Microscopic rules -> Mesoscopic structure

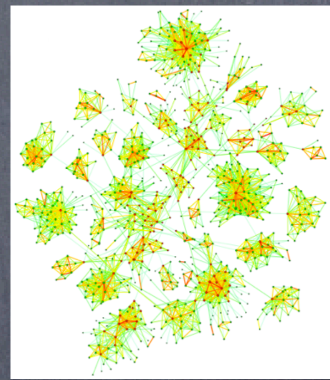
Topology

Topology & weights

Macroscopic



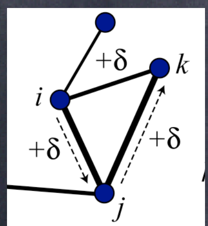


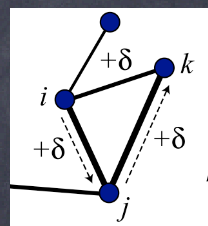


$\delta = 0$

$\delta > 0$

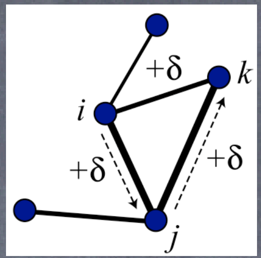
Microscopic

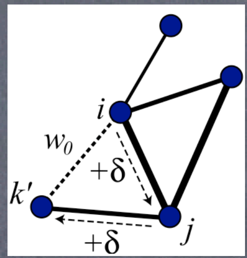




Microscopic rules in the model

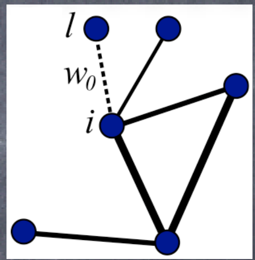
- Local attachment (LA)





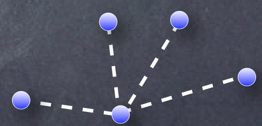
- Global (random) attachment (GA)

$$k_i = 0 \implies P(i,j) = 1; w_{ij} = w_o = 1$$
$$k_i > 0 \implies P(i,j) = p_r; w_{ij} = w_o$$



- Node deletion (ND)

$$k_i > 0 \implies P(k_i = 0) = p_d$$

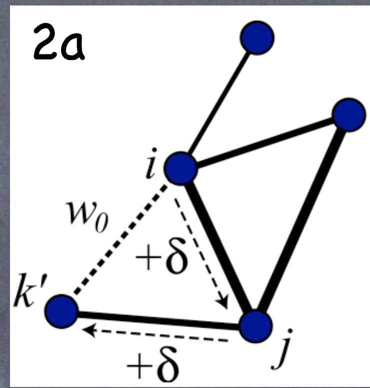


Microscopic rules in the model

- Local attachment (LA)

(1) Weighted local **search** / reinforcement

$$P(i \to j) = w_{ij}/s_i$$
$$P(j \to k) = w_{jk}/(s_j - w_{ij})$$
$$w_{ij} \to w_{ij} + \delta$$
$$w_{jk} \to w_{jk} + \delta$$

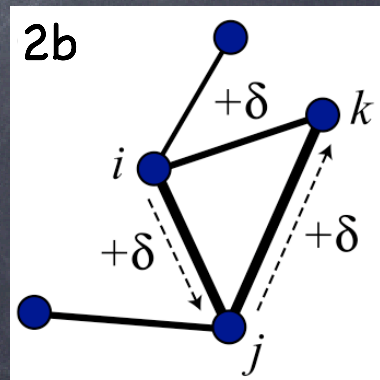


(2a) If (i,j,k) does not exist => **Triangle formation**

$$P(i,j,k) = p_\Delta$$
$$w_{ik} = w_0 = 1$$

(2b) If (i,j,k) exists => **Triangle reinforcement**

$$w_{ik} \to w_{ik} + \delta$$



Microscopic rules in the model

- Summary of the model
  - Weighted local search for new acquaintances
  - Reinforcement of popular links & Triangle formation
  - Unweighted global search for new acquaintances
- Parameters

δ	Free weight reinforcement parameter
$p_d = 10^{-3}$	Sets the time scale of the model $\langle \tau_N \rangle = p_d^{-1}$
$p_r = 5 \times 10^{-4}$	Global connections; Not sensitive
p_Δ	Adjusted w.r.t. δ to keep $\langle k \rangle$ constant

Model mechanisms vs. sociology

Network sociology*

- Cyclic closure
  - Exponential decay
- Focal closure
  - Independent of distance
- "Sample window"

Model

- Local attachment (LA)
- Global attachment (GA)
- Node deletion (ND)

* M. Kossinets et al., "Empirical Analysis of an Evolving Social Network", Science **311**, 88 (2006)

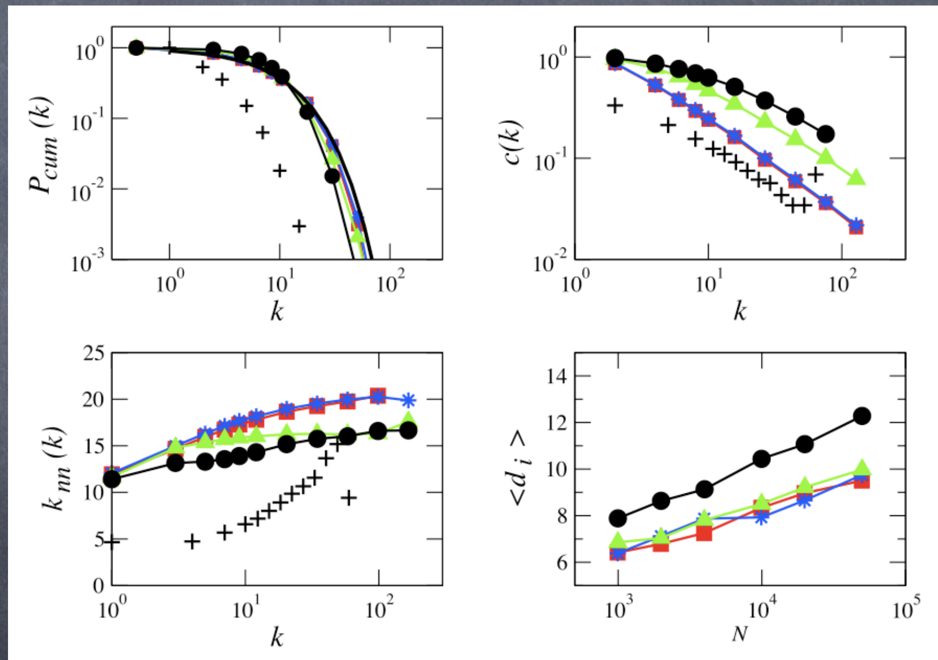
Basic characteristics

(a) Fat-tailed degree distribution

(b) High clustering

(c) Assortative

(d) Small world

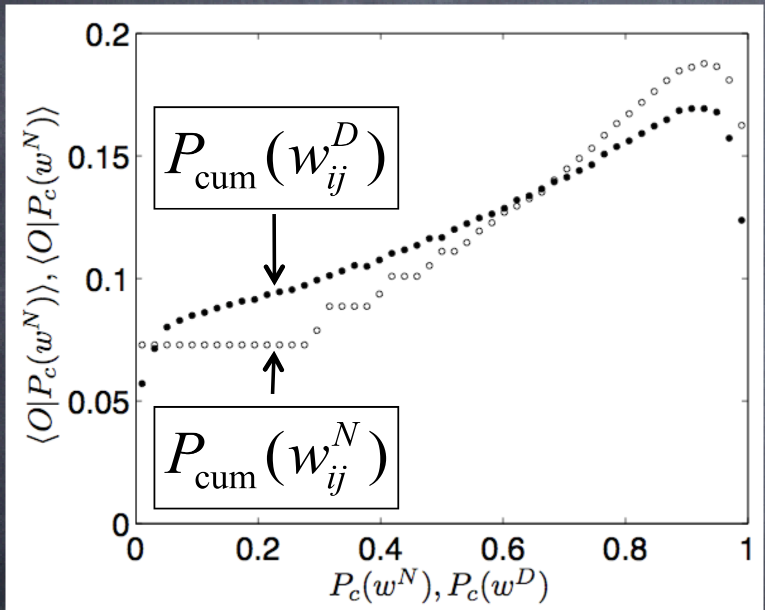


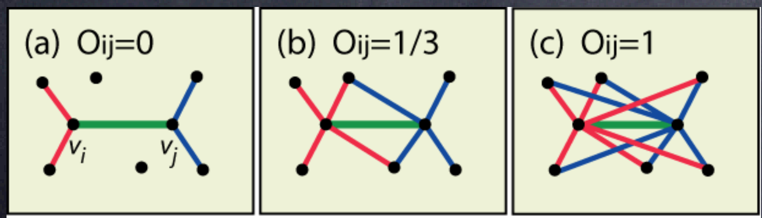
- $\delta = 1$
- $\delta = 0.5$
- $\delta = 10^{-3}$
- $\delta = 0$

alizations of $N = 5 \times 10^4$ networks. Values of δ are 0 ($\square$), 1×10^{-3} ($*$), 1×10^{-2} ($\triangleright$), 0.1 ($\triangle$), 0.5 ($\triangledown$), and 1 ($\circ$).

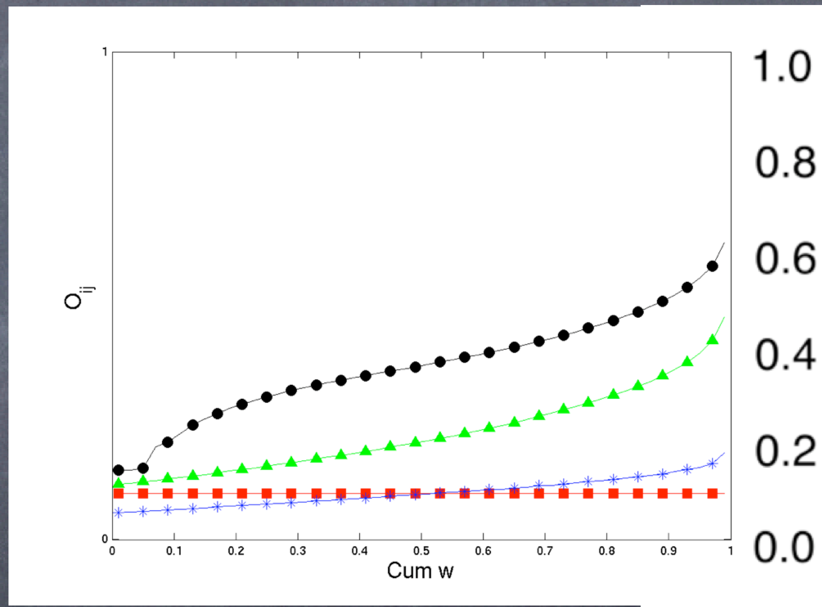
Local structure

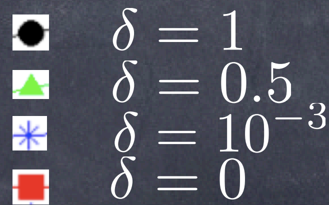
Empirical





Model





Global structure

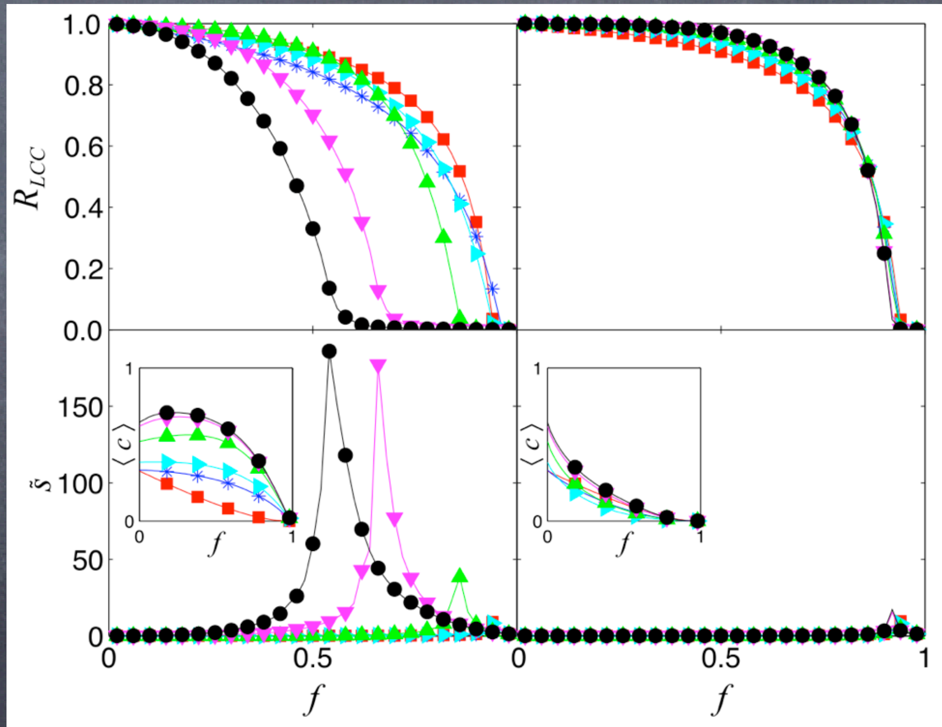
- **Weak ties hypothesis (WTH)***: implies weight-topology correlations: Ties within communities are strong, ties between communities are weak
- Explore weight-topology correlation with link percolation
- Control parameter $f \in [0, 1]$
- Order parameter $R_{\mathrm{LCC}} \in [0, 1]$

*M. Granovetter, "The Strength of Weak Ties", The American Journal of Sociology **78, 1360** (1973)

Global structure

- Small $\delta < 0.1$ — $\delta = 10^{-3}$, $\delta = 0$
  - Network disintegrates at the same point for weak/strong link removal
  - Incompatible with WTH
- Large $\delta > 0.1$ — $\delta = 1$, $\delta = 0.5$
  - Network disintegrates at different points
  - WTH compatible community structure

Weak go first | Strong go first



alizations of $N = 5 \times 10^4$ networks. Values of δ are 0 (□), 1×10^{-3} (∗), 1×10^{-2} (▷), 0.1 (△), 0.5 (▽), and 1 (○).

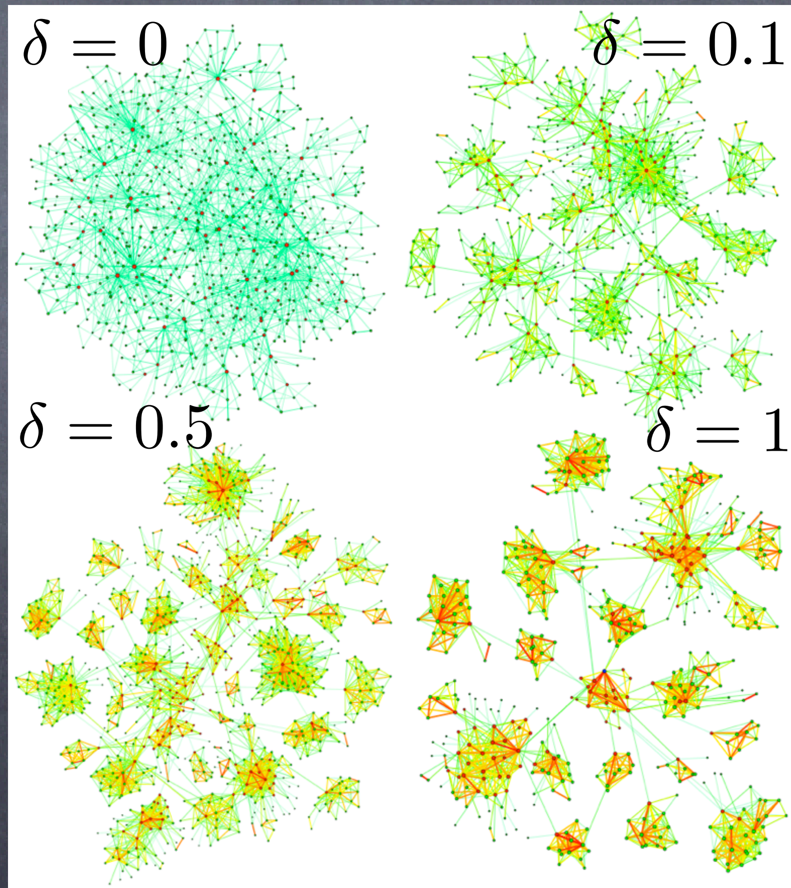
Communities by inspection

- Average number of links constant $\langle L \rangle = N\langle k \rangle/2$
  => All changes in structure due to reorganisation of links

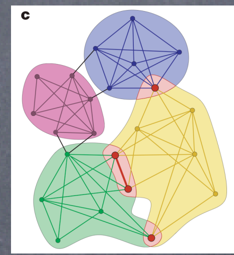
- Increasing δ traps walks in communities, further enhancing trapping effect

  => Clear communities

- Triangles accumulate weight and act as nuclei for communities

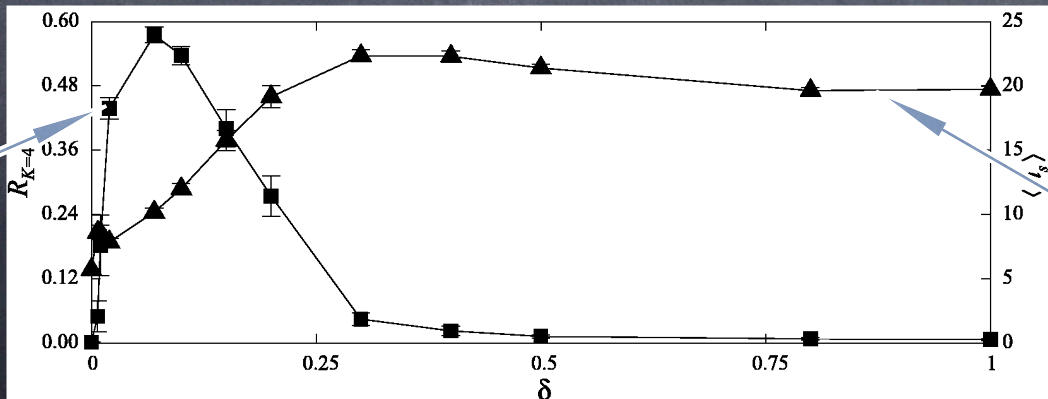


Communities by k-clique method



- Use k-clique algorithm / definition for communities*
- Focus on 4-cliques (smallest non-trivial cliques)
  - Relative largest community size $R_{k=4} \in [0, 1]$
  - Average community size (excl. largest) $\langle n \rangle$
- Observe clique percolation through the system for small δ
- Increasing δ leads to condensation of communities

$R_{k=4}$



$\langle n \rangle$

* G. Palla et al., "Uncovering the overlapping community structure...", Nature **435**, 814 (2005)

Is community size distribution stable?

- Consider community k with size N_k
- In the large δ regime, most local random walks remain in the initial community, resulting in stable distribution

$$\frac{\mathrm{d}N_k}{\mathrm{d}t} = -p_d N_k + p_d N \frac{N_k}{N} = 0$$

Rate of deleting nodes **within** the community

Rate at which **new nodes** will join the community during subsequent LA steps

- Community formation happens in transient state
- A triangle accumulating weight acts as a nucleus for the emerging community

Conclusion

- Local coupling between network topology and tie strengths (WTH)
- Weak ties (PT) are qualitatively different from strong ties (no PT)
- Model: essential characteristics & local & global properties
- Need focal & cyclic closure & sufficient reinforcement of connections
- Communities result from initial structural fluctuations that become amplified by repeated application of the microscopic processes

References

- J.-P. Onnela, J. Saramäki, J. Hyvönen, G. Szabó, D. Lazer, K. Kaski, J. Kertész, and A.-L. Barabási, "Structure and tie strengths in mobile communication networks", PNAS **104**, 7332 (2007).
- J. M. Kumpula, J.-P. Onnela, J. Saramäki, K. Kaski, and J. Kertész, "Emergence of communities in weighted networks" Phys. Rev. Lett. **99**, 228701 (2007).
- See also Science **314**, 914 (2006).
- See http://www.physics.ox.ac.uk/users/Onnela/

THANK YOU!